



## Application of Garch Model in Forecasting Volatility of Exchange Rates

<sup>1\*</sup>Ibrahim, H, <sup>2</sup>Rasheed, B.A. and <sup>2</sup>Abdulkarim, K.

<sup>1</sup>Department of Statistics, Jigawa State Polytechnic, Dutse, Nigeria

<sup>2</sup>Department of Mathematics and Statistics, Federal Polytechnic, Bauchi, Nigeria

### ABSTRACT

A foreign exchange transaction is an agreement between a buyer and a seller that a given amount of one currency is to be delivered at a specified rate for other currencies in the foreign exchange market. Volatility forecasting for multi-state ahead forecast in a time series amounts to forecast a sequence of future values based on the observed previous values. Examples of time series for which multi-step ahead forecast is useful include crop yield, stock price, and electrical power consumption. However, to achieve this aim, we employ GARCH model to determine the best order to forecast the exchange rate in the foreign exchange market. However, it was concluded that the GARCH has the least Akaike Information Creation (AIC) among the selected orders which were based on the foreign exchange standard plots.

**Keywords:** Garch model, Heteroscedasticity, exchange rate and Forecasting.

### INTRODUCTION

The foreign exchange market provides the physical and institutional structure through which the money of one country is exchange for that of another country. The rate of exchange between currencies is determined and foreign exchange transaction are physically completed. Stock market volatility is a measure of the variance of the price of a financial asset over time. Market volatility is critical to investors since it provides information on the uncertainty of an asset (Hongyu and Zhichao, 2006). The volatility of an asset guides investors in their decision making process because investors are interested in returns and their uncertainty (Arestis *et al*, 2000; Mala and Reddy, 2007).

Geographically, the foreign exchange market spans the globe with prices moving and currencies traded somewhere every hour of every business day. In some countries, a portion of foreign exchanged is conducted in an official trading floor by open bidding, closing prices are published as the official price of “fixing” for the day and certain commercial and investment transactions are based on this official price (Adam *et al.*, 2021).

The time series data may possess properties like hetroscedasticity, heavy tails, leptokurtic distribution, volatility clustering, absence of autocorrelations and leverage effect. The Autoregressive Conditional Heteroskedasticity

(ARCH) model with normal innovations first introduced by Engle (1982) captured some of stylized characteristics of time series data. Bollerslev (1986) further introduced additional parameter(s) of the ARCH model to generalized ARCH model (GARCH) to capture the phenomenon earlier mentioned in time series data. However, traditionally, time series were modeled with normal errors. Unfortunately, such models still failed to sufficiently capture the main stylized characteristics of some time series data, such as the heavy tails, leptokurtic and skewness. A number of researches have investigated the performance of GARCH models with non-normal error distribution. For example, Hansen (1994) considered a GARCH model with skewed-student-t distribution to capture the skewness and the excessive kurtosis. Liu and Hung (2010), and Bali (2007) proposed GARCH models with skewed generalized error distribution (SGED). Gokcan (2000) compared the performance of volatility forecasting of GARCH model versus EGARCH model using the monthly stock market returns of seven emerging countries. It was found that the GARCH model outperforms the EGARCH model, even if the stock market return series exhibit skewed distributions.

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Address Correspondence to:

[kuyes2000@gmail.com](mailto:kuyes2000@gmail.com)

However, the assumption of homoscedasticity permits the use of the estimated regression equation to make forecast of the dependent variable. However, practically, all time series data tend to exhibit varying variance; which vitiates the homoscedasticity assumption. Therefore, it becomes appropriate to consider frameworks that allow the variance to depend on its history. Indeed, a robust estimation should be considered when there is a strong suspicion of heteroscedasticity.

In the homoscedastic model, it is assumed that the variance of the error term is constant for all values of  $x$ . Heteroscedasticity allows the variance to be dependent on  $x$ , which is more common for many real scenarios. For example, the variance of the foreign exchange might be different from time to time. This study therefore evaluated some heteroscedastic models in fitting and forecasting data on foreign exchange over time.

## METHODOLOGY

This study will consist of detail description of the analyses of time series data on Nigeria foreign exchange which consists of official and bureau-de-change exchange rates. The time period of the analyses is from January 1999 to December 2020 for official exchange rate, which makes up 240 observations. However, in bureau-de-change, data was collected from January 2004 to December 2020. In this research, models namely, GARCH will be selected to capture the data over time.

### Augmented Dickey – Fuller Test

According to (Shumnay and Stoffer, 2011), the ADF test is used to test for unit root, the testing procedure for the ADF test is the same as for the Dickey-Fuller test but it is applied to the model. A random walk with drift and trend is represented as;

$$\Delta Y_t = \alpha + \beta_t + \gamma Y_{t-1} + \sigma_1 \Delta Y_{t-1} + \dots + \sigma_{p-1} \Delta Y_{t-p+1} + \epsilon_t \quad (1)$$

Where  $\alpha$  is a constant,  $\beta$  the coefficient on a time trend and  $P$  the lag order of the autoregressive process. Imposing the constraints  $\alpha = 0$  and  $\beta = 0$  corresponds to modelling a random walk and using the walk with a drift.

The test statistic, value is calculated as follows:

$$t = \frac{\hat{\gamma}}{\sigma_{\hat{\gamma}}} \quad (2)$$

where  $\hat{\gamma}$  is the estimated coefficient and  $\sigma_{\hat{\gamma}}$  is the standard error in the coefficient estimate.

$H_0: \gamma = 0$  Vs  $H_1: \gamma < 0$

If  $t^* >$  ADF critical value = not reject null hypothesis, that is, unit root exists.

If  $t^* <$  ADF critical value = reject null hypothesis, that is, unit root does not exist. Using the usual 5% threshold, differencing is required if the  $p$  – value is greater than 0.05. (Shumnay and Stoffer, 2011).

### Kwiatkowski, Phillips, Schmidt and Shin test

(Kwiatkowski, Phillips, Schmidt and Shin 1992) proposed an LM test for testing trend and/or level stationarity (the KPSS test). That is, now the null hypothesis is a stationary process. Taking the null hypothesis as a stationary process and the unit root as an alternative is in accordance with a conservative testing strategy. Hence, if we then reject the null hypothesis, we can be pretty confident that the series indeed has a unit root. Therefore, if the results of the tests above indicate a unit root but the result of the KPSS test indicates a stationary process, one should be cautious and opt for the latter result

Null hypothesis

$$H_0: \sigma_e^2 = 0$$

Under the null hypothesis of  $e_t \sim NIID(0, \sigma_e^2)$ , the test statistic is

$$LM = \frac{\sum_{t=1}^T S_t^2}{\hat{\sigma}_e^2} \quad (3)$$

$$\hat{\sigma}_e^2 = \frac{\sum_{t=1}^T e_t^2}{T} \quad (4)$$

$$S_t = \sum_{i=1}^t e_i, t = 1, \dots, T \quad (5)$$

where  $e_t$  are the residuals from the regression of  $y_t$  on a constant and a time trend.

### Analytical Framework of GARCH Model.

Every GARCH family model requires two distinct specifications: the mean and variance equations. According to Engel, conditional heteroscedasticity in time series denoted by  $y_t$  can be modeled using ARCH model expressing the mean equation in the form:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_p y_{t-p} + \epsilon_t = \alpha_0 + \sum_{i=1}^p \alpha_i y_{t-i} + \epsilon_t \quad (6)$$

Equation 6 is a ‘ $p$ ’ order autoregressive model. The mean equation is the same for all ARCH/GARCH family models.

Where  $\alpha_0 > 0$   $\epsilon_t = \sim N(0, \sigma^2)$ . the residual ( $\epsilon_t$ ) is independent and identically distributed with a mean of zero and unit variance.

**Generalized Autoregressive Conditional Heteroscedastic (GARCH) Model**

Bollerslev (1986) and Taylor (1986) Model If an autoregressive moving average model (ARMA model) is found for the error variance ( $\sigma_t^2$ ), the model is a generalized autoregressive conditional heteroskedasticity (GARCH) model. The general specification of GARCH (p, q) is as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_q \epsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_p \sigma_{t-p}^2 = \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 \tag{7}$$

Where  $\sigma_t^2$  will be replaced by  $h_t$

Where  $\sigma_t^2$  is the conditional variance and  $\epsilon_{t-i}^2$  is the past squared residual return also  $\alpha_0 > 0$ ,  $\alpha_i \geq 0$ ,  $\beta_j \geq 0$  the above is GARCH(p,q) model.

Where  $\alpha_i$  = coefficients of the ARCH component; and  $\beta_i$  = coefficient of the GARCH component. The three parameter ( $\alpha_0$ ,  $\alpha_i$  and  $\beta_i$ ) are restricted to be positive (assumed to be non-negative) and  $\alpha_i + \beta_i < 1$  to achieve stationarity

The test for the present of GARCH effect is established in two steps: First, we estimate the best fitting AR(q) model which is the AR (q) model with a lag order that gives the lowest Aikaike and Swartz information and highest log-likelihood ratio.

Compute and plot the autocorrelations of  $\hat{\epsilon}_t^2$  by

$$\frac{\sum_{t=i+1}^T (\hat{\epsilon}_t^2 - \hat{\sigma}_t^2)(\hat{\epsilon}_{t-1}^2 - \hat{\sigma}_{t-1}^2)}{\sum_{t=1}^T (\hat{\epsilon}_t^2 - \hat{\sigma}_t^2)^2} \tag{8}$$

**RESULTS AND DISCUSSION**

**Presentation of the Differenced Data**

Figures 1 and 2 shows the plot of the official and non-official exchange rates respectively after taking the first difference.

Figure 3 to 6 show the correlellograms of both exchange rates. The charts shows that the two data series are stationary. This is most clearly seen in the ACF charts for both data. Note also that the main feature of the correlellogram for data to be stationary, is that, the autocorrelation tends towards zeros as the lags increases, as can be seen from the charts.

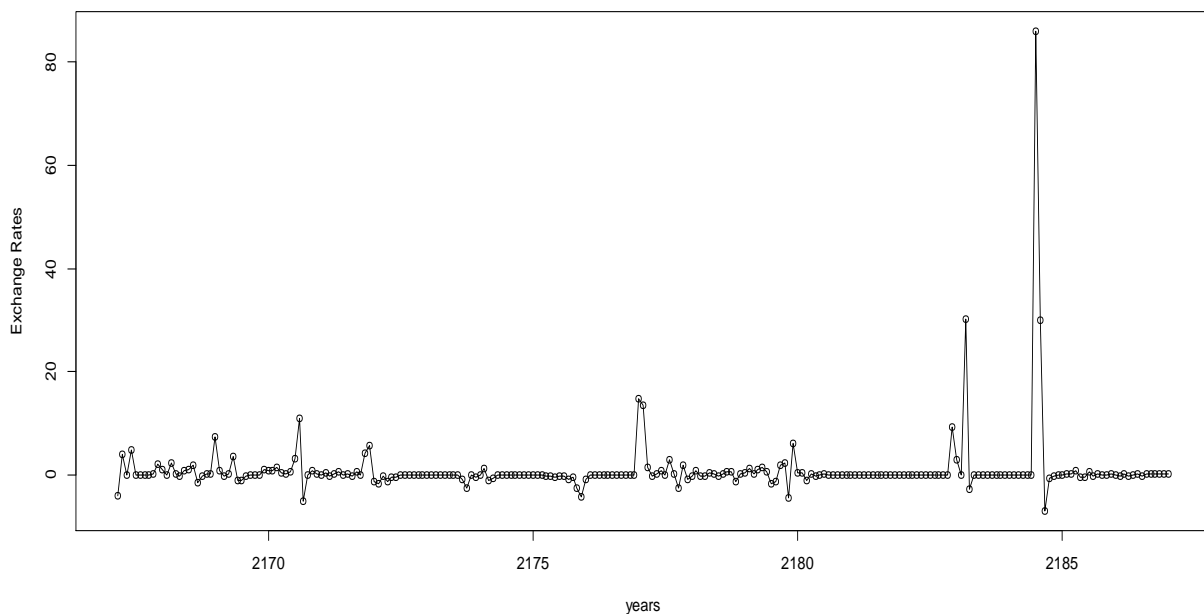


Fig.1: Differenced official exchange rate

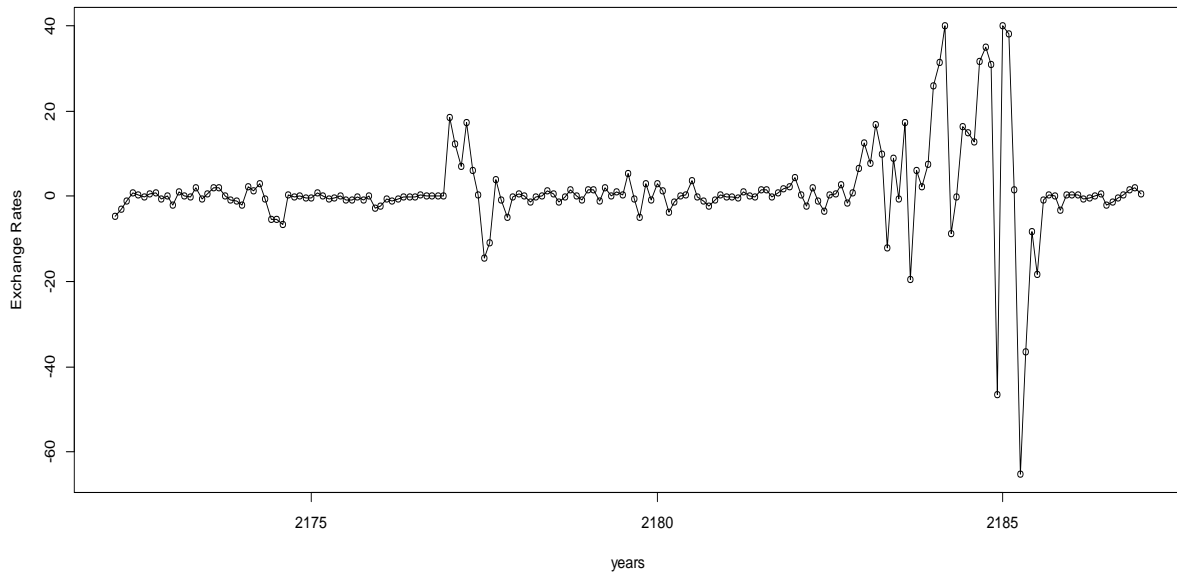


Fig.2: Differenced non-official exchange rate

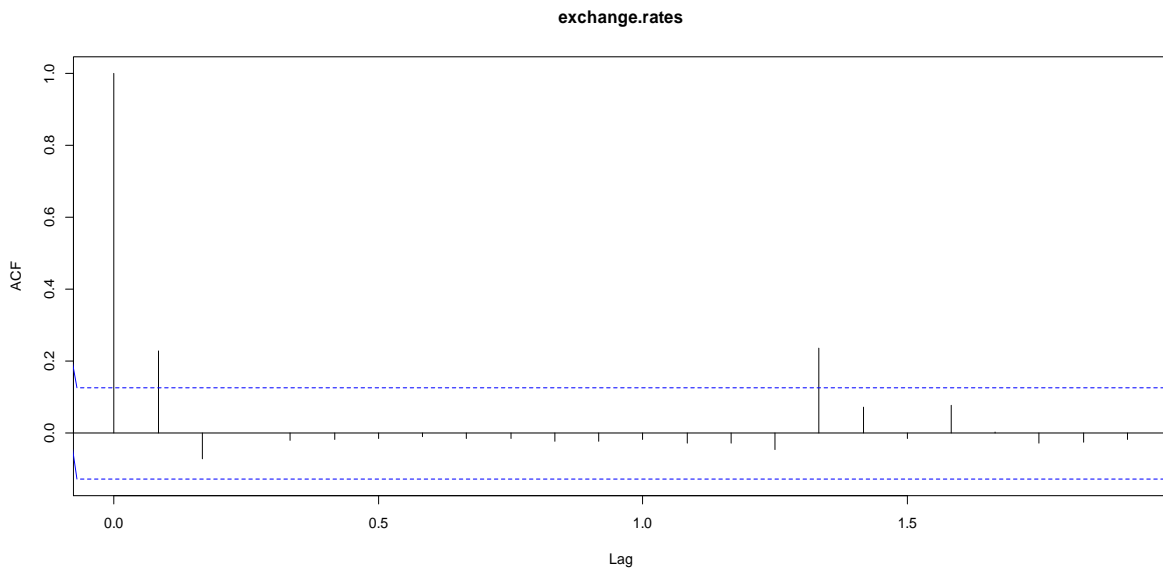


Fig. 3: ACF plot for the differenced official exchange rate

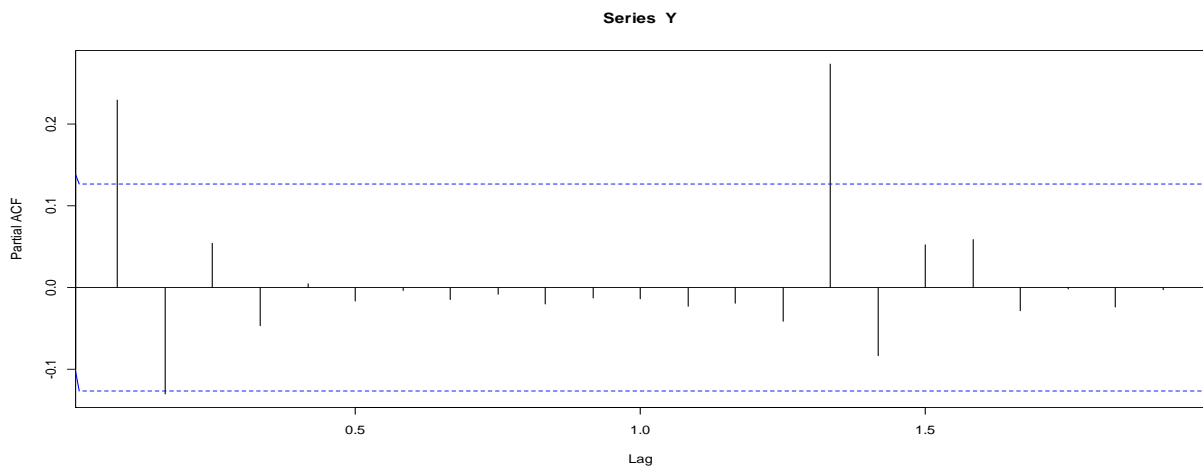


Fig. 4: PACF plot for the differenced official exchange rate

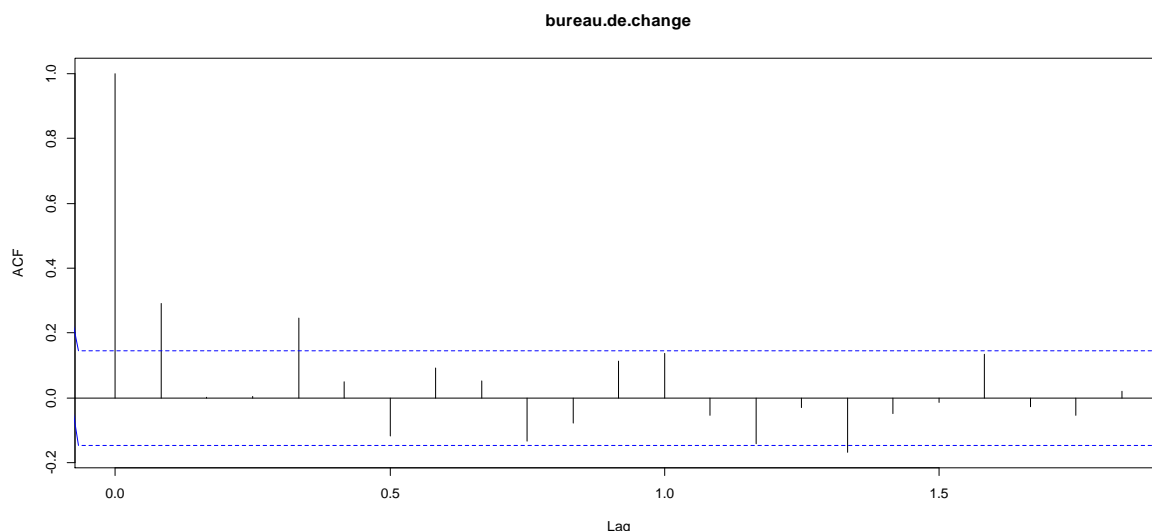


Fig. 5: ACF plot for differenced Non-official exchange rate

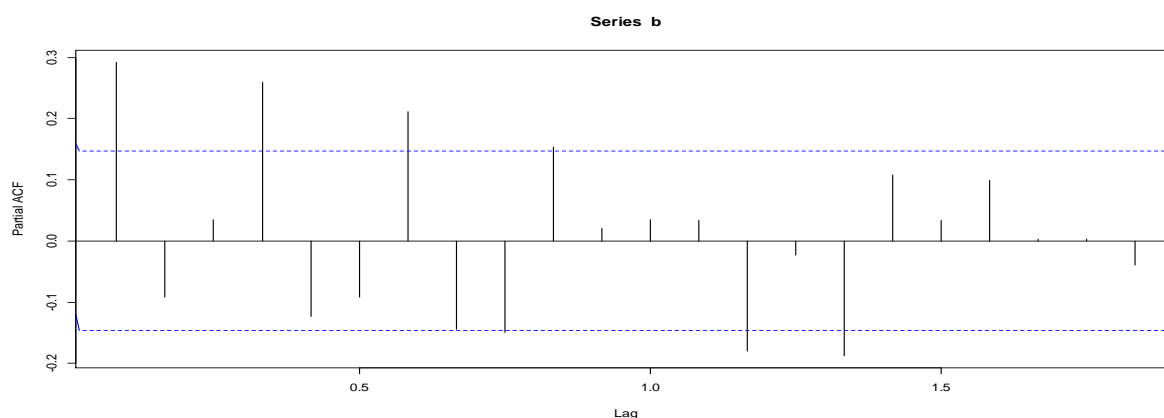


Fig. 6: PACF plot for differenced Non-official exchange rate

**Testing for Unit Root and Stationarity in the Differenced Foreign Exchange Data**

Table 1a: Stationary test of the Official exchange rate

Test statistics	Test values	Lag order	p-value	Hypothesis	Decision	Remark
ADF	-5.9007	6	0.01	Unit root	Reject $H_0$	Stationary
KPSS	0.2241	4	0.1	Unit root	Accept $H_0$	Stationary

Table 1b: Stationary test of the Non-official exchange rate

Test statistics	Test values	Lag order	p-value	Hypothesis	Decision	Remark
ADF	-5.1158	6	0.01	Unit root	Reject $H_0$	Stationary
KPSS	0.1978	4	0.1	Unit root	Accept $H_0$	Stationary

The stationary unit root test was carried out using Augmented Dickey Fuller (ADF) and Kwiatkowski Phillip Schmidt shin (KPSS). The statistic tests the null hypothesis that the data series has a unit root with the alternative that the

data series is stationary for ADF. While KPSS reverse is the case. The results obtained for the differenced official and differenced Non-official are presented in table 1a and 1b respectively

which also indicated that data are stationary after the first difference.

Table 1a presents the stationarity tests for the differenced official exchange rate over the period of investigation with a null hypothesis of a unit root against alternative hypothesis of a level of stationarity for ADF and KPSS. The p-values of 0.01 for ADF is less than the 5% level of significance while p-value 0.1 for KPSS is greater than the 5% level of significance, which indicate that, the null hypothesis of having a unit root series should be rejected in favour of alternative of being stationary and vice versa for KPSS. Indeed, the data is stationary after the first difference hence we can proceed to fitting and forecasting of the series.

Table 1b presents the stationarity tests for the differenced Non-official rate over the period of investigation with a null hypothesis of a unit root against alternative hypothesis of a level of stationarity for ADF and PP and vice versa for KPSS. The p-values of 0.01 both for ADF and PP are less than the 5% level of significance while p-value 0.1 for KPSS is greater than the

5% level of significance, which indicate that, the null hypothesis of having a unit root series should be rejected in favour of alternative of being stationary and vice versa for KPSS. Indeed, the data is stationary after the first difference hence we can proceed to fitting and forecasting of the series.

#### Testing for Heteroscedasticity in the Data

The test of heteroscedasticity for the data across the locations at different time lags were carried out using Breusch-Pagan test. The Breusch-Pagan tests the null hypothesis of no heteroscedasticity against its alternative of there is heteroscedasticity. The Table 4.5 below shows the detail of the analysis.

It was observed that the p-values of the statistic from the four form of the data are less than the critical value of 0.05 and we therefore reject the null hypothesis of data being homoscedastic in favor of alternative of being heteroscedastic. Indeed, the test confirms that the data series of the two foreign exchange in its original and differenced form.

Table 2: Testing for heteroscedasticity in the data.

Exchange rate type	Test values	DF	p-value	Hypothesis	Decision	Remark
Official (Original)	9.668	4	0.0018	No Heteroscedasticity	Reject ( $H_0$ )	data is heteroscedastic
Non-official (Original)	15.72	4	0.0034	No Heteroscedasticity	Reject ( $H_0$ )	data is heteroscedastic
Official (diff)	10.831	4	0.0285	No Heteroscedasticity	Reject ( $H_0$ )	data is heteroscedastic
Non- Official (diff)	11.177	4	0.0246	No Heteroscedasticity	Reject ( $H_0$ )	data is heteroscedastic

#### Fitting the GARCH Model

Different orders of GARCH are fitted to the stationary differenced data for both official and bureau-de-change exchange rates. The parameter, AIC and BIC of each order of the models are displayed in table 4.6 below. The model with minimum criteria is considered as the best and therefore the so order of the model is best to capture the exchange rates.

With regard to the parameters reported in tables 3a and 3b above, The estimated coefficient

values of all GARCH (p,q) strictly conforms to the bounds of parameter, between -1 and 1 except GARCH (1,1) which is out of the bound. This has made the models to be stationary. Additionally, comparing the GARCH models above in terms of the AIC and BIC, GARCH(1,1), is the best for both local and non-official since their estimated AIC and BIC are smaller as compared to other models. Indeed, based on the parameter estimates and the criteria, GARCH(1,1) is chosen as the best model to capture both foreign exchange data.

Table 3a: Fitting the GARCH(p, q) for official price

Model	$\mu$	$\omega$	$\alpha_1$	$\alpha_2$	$\beta_1$	$\beta_2$	AIC	BIC
(1,1)	128.418	-11.0082	-0.2623	-	5.6590	-	8.579	8.637
(1,2)	1.284e+02	8.031e-01	8.923e-01	-	1.545e-01	1.000e-08	8.590	8.662
(2,1)	1.284e+02	9.677e-01	8.601 e-01	1.9373e-01	1.000e-08	-	8.588	8.661
(2,2)	1.284e+02	9.677e-01	8.601e-01	1.933e-01	1.000e-08	1.000e-08	8.596	8.683

Table 3b: Fitting the GARCH(p, q) for non-official price

Model	$\mu$	$\omega$	$\alpha_1$	$\alpha_2$	$\beta_1$	$\beta_2$	AIC	BIC
(1,1)	160.054	1.9196	0.9011	-	0.1642	-	9.036	9.106
(1,2)	1.534e+02	5.404e-01	1.000e+00	-	1.489e-03	9.068e-02	9.103	9.192
(2,1)	1.600e+02	1.910e+00	8.927e-01	1.000e-08	1.667e-01	-	9.060	9.149
(2,2)	1.599e+02	1.714e+00	9.391e-01	1.000e-08	1.000e-08	1.030e-01	9.047	9.154

### Model Adequacy (Diagnostic) checking of estimated models (Standardized Residuals Tests)

Table 4a: GARCH Model Diagnostic with Respect to the official exchange rate

Model	GARCH(1,1)		GARCH(1,2)		GARCH(2,1)		GARCH(2,2)	
	Values	P-value	Values	P-value	Values	Pvalue	Values	P-value
Jarque-Bera Test	11.246	0.0361	1.2333	0.5397	1.4739	0.4785	1.4739	0.4785
Shapiro-Wilk Test	11.246	0.0361	1.2333	0.5397	1.4739	0.4785	1.4739	0.4785
Ljung-Box Test ( $R^2$ )	0.8000	0.000	0.8013	0.000	0.8016	0.000	0.8016	0.000
LM Arch Test	11.981	0.0286	12.087	0.2792	13.782	0.1831	13.782	0.1831

Table 4b: GARCH Model Diagnostic with Respect to the non-official exchange rate

Model	GARCH(1,1)		GARCH(1,2)		GARCH(2,1)		GARCH(2,2)	
	Values	P-value	Values	P-value	Values	Pvalue	Values	P-value
Jarque-Bera Test	15.281	0.00048	12.380	0.0020	15.191	0.0005	18.354	0.0001
Shapiro-Wilk Test	0.8651	1.37e-13	0.8587	6.6156e-12	0.868	1.923e-11	0.863	1.075e-11
Ljung-Box Test ( $R^2$ )	29.929	0.071	8.4358	0.5863	23.752	0.0082	29.116	0.0011
LM Arch Test	15.359	0.2223	9.945	0.6208	15.388	0.2208	20.298	0.0616

Having carried out the fitting comparison analysis, and the GARCH (1,1) is chosen as the best or tentative models as opposed to others, based on the conclusion in Table 4.6 above, the adequacy of the chosen model is further tested

to draw empirical conclusions regarding the model as good fit. These tests carried out are Ljung-Box, normality test of the residuals using Shapiro-Wilk normality test statistic and LM Arch Test. The results are reported in Table 4.

The Table 4a and 4b shows the Ljung-Box and Jarque-Bera tests for the two exchange rate type with chi-square statistics that give corresponding p-values. The tests are significant for GARCH (1,1), therefore, the residuals appear to be uncorrelated. This indicates that the residuals of the fitted GARCH (1,1) model are white noise, as such, the model fits the series quite well (the parameters of the model are significantly different from zero), so we can use this model to make forecasts. Ljung-Box Test ( $R^2$ ) of GARCH (1,1) indicates that all the past observations of both exchange rate contribute significantly to the present response and the model is well fitted. Furthermore, the normality is not significant; hence, the Shapiro-Wilk test suggests that the standardized residuals are normal. This also supports the fact that the residuals of the fitted GARCH models are white noise, and the model fits the series quite well (since one the assumptions of the residual being white noise is normality). Hence the model is stationary due to the presence of white noise. The p-values of LM Arch Test also indicate the fitted model capture the heteroscedasticity in the data over time'.

## Conclusion

In this study, comparative performance of the GARCH model different orders was carried out on foreign exchange data. It can be seen that, because of the volatile nature of the foreign exchange data, the data was heteroscedastic. In the comparative performances of the models, The estimated coefficient values of all GARCH (p,q) strictly conform to the bounds of parameter, between -1 and 1. This has made the models to be stationary. Meanwhile, comparing the GARCH models using the selected criteria, GARCH (1,1) obviously preferred to be the best model that captured the both exchange rate over time.

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